

State-of-the-Art and Future Directions for Predictive Modelling of Offshore Structure Dynamics using Machine Learning

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ABSTRACT

Ramboll Oil and Gas are leading the field in the development of Structural Health Monitoring Systems (SHMS) for offshore structures. This paper outlines the State-of-the-Art process for predictive maintenance that Ramboll have developed and implemented for offshore structures. This system is one of the first, if not the only one, that creates a maintenance schedule based on knowledge of the structure's current state.

The State-of-the-Art methods of today, as adopted by Ramboll, encompass advanced analysis methods ranging from linear and non-linear system identification, expansion processes, Bayesian FEM updating, wave load calibration, quantification of uncertainties from measured data, damage detection and structural re-assessment analysis to Risk- and Reliability-Based Inspection Planning (RBI) analysis.

The paper will be the first in a series of papers that will outline various promising methods contributing to an even better understanding of the issues at stake in the offshore structures context.

Keywords: Linear and Nonlinear System Identification, FEM Updating, Modal Expansion, Wave Load Calibration, Digital Twin, Uncertainties, Predictive Maintenance, Machine Learning, Grey-Box.

INTRODUCTION

Close to 600 offshore oil and gas platforms in the North Sea have either already exceeded their design lifetime or are currently approaching the end of their life within the next few years. The industry is facing huge investments in infrastructure in the near future for maintaining today's oil and gas production. In Denmark alone, investments for a major upgrade/reinforcement of the Danish oilfield infrastructure have been estimated at a cost of close to \$5 billion for maintaining Danish oil and gas production in the future. The alternative to upgrading of the fields and reducing production is decommissioning of the platforms, and the cost for shut down and removing the platforms is estimated at \$2.5 billion. The availability of reliable and enhanced methods to provide the information for decision making is of paramount importance for economic profitability, environmental protection and human safety for future operation.

Recent reviews performed by the International Oil & Gas Producers Association [1] of the industry methods and practices for structural design and re-assessment reveal potential inconsistencies in the methods adopted and even knowledge gaps. All structures shall be operated within required safety levels as specified by codes and standards, so it is of the utmost importance that the real uncertainties associated with the design methods are correctly quantified, so that the risks of operating the platform are controlled throughout the lifetime of the structure, either for the original design, for the reinforced/upgraded design, or for other extended lifetime activities performed for ageing platforms.

Throughout the past 15 years, Ramboll have developed novel methods for assessment of the real/measured uncertainties associated with the design and analysis of platforms and today Ramboll is considered to be among the leading experts within application of Structural Health Monitoring Systems (SHMS) for condition-based lifetime extension and for quantification of the real model prediction uncertainties. The uncertainties form the basis for assessing the time dependent Probability of Failure (PoF) for the platforms, i.e. the safety level of the platforms, and hence are the key factor for both verification of extended lifetime of the platforms,

but also for reduction of maintenance costs during the years of production. Reductions in maintenance costs are achieved by linking the quantification of measured uncertainties with Risk- and Reliability-based Inspection Planning (RBI) methods. Projects on large offshore structures have been performed for which measured site specific uncertainties have formed the basis for RBI with the result of considerable cost savings for future platform maintenance cost.

RAMBOLL PREDICTIVE MAINTENANCE

An overview of the Ramboll Predictive Maintenance Process is shown in Fig. 1, the process can be split into two stages. The first is the calibration of a Finite Element (FE) model with regard to the measured data from the structure of interest. The second stage is where decisions, as to the maintenance and operation of the structure, are made.

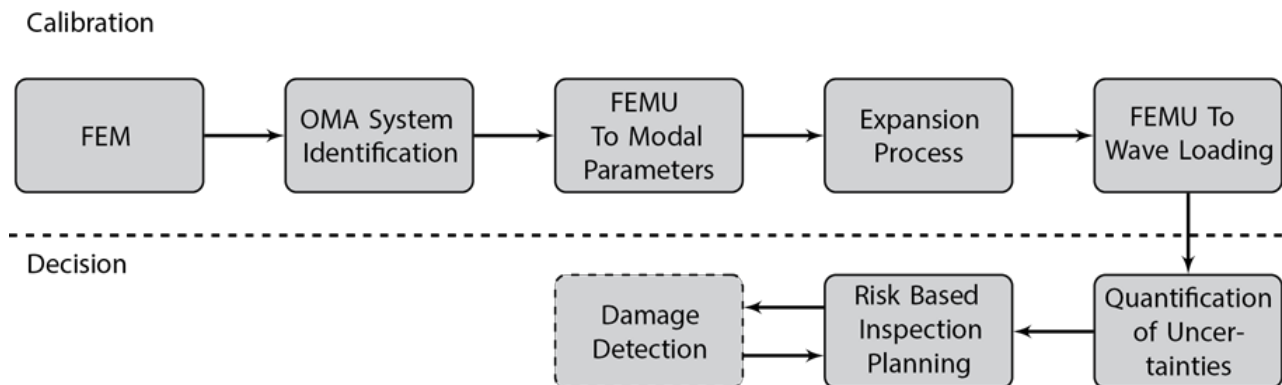


Fig. 1 Flow diagram giving a top level view of the Ramboll Predictive Maintenance Process, the process of data to decision can be clearly seen in the methodology

The calibration phase of the process can be seen as the combination of two Finite Element Model Update (FEMU) steps. The beginning of the calibration method is to build a high fidelity FE model, which is achieved using Ramboll's ROSAP package [2]. After a period of data acquisition on the structure including measurement of accelerations, strains, wave characteristics (via wave radar), and GPS measurements; the modal parameters of the structure can be determined using the Stochastic Subspace Identification (SSI) method for Operational Modal Analysis (OMA).

The first FEMU step is the update of the FE model to the measured modal parameters based on a sensitivity analysis and prior information in the form of a Bayesian parameter update. Once satisfied that the FE model sufficiently represents the dynamic behaviour of the structure, a modal expansion process can be used to determine the strains/stresses across the structure. A compensation has to be made for the quasi-static response of the structure with relation to the wave loading. The predictions obtained from the expansion process and the measured sea states are then used in the second round of FEMU to calibrate the parameters of the wave loading.

Throughout the calibration process, it is possible to quantify the uncertainty introduced in the model, the measurements, and the updating steps with respect to fatigue damage accumulation experienced by the structure. The predictions of the fatigue damage accumulation and their uncertainties, summarised as a bias value and a Coefficient of Variation (CoV) are used in a risk based inspection planning (RBI) system. The addition of online damage detection to this is a key step towards a robust predictive maintenance system informed by data. This forms the decision half of the process which is not discussed in full in this work, rather, insight to the calibration methods are presented.

The key first step to providing a predictive maintenance strategy for Ramboll's clients is the acquisition of high-fidelity, information-rich data. Typically a Structural Health Monitoring System (SHMS) used by Ramboll will be based on a range of sensors, such as accelerometers, strain gauges, GPS's, anemometers and wave radars of different types.

Initial work has been carried out to ensure that the base data logged from monitoring systems are of high quality. A number of activities for quality assurance of the data collected are performed, ranging from statistical interrogation to further improvement of data accuracy of platform displacement data. Additionally, algorithms for combining signals from GPS and accelerometers for improved displacement measurements [3], laboratory experiments [4], and full scale experiments on offshore structures [5] are used to validate the principles used.



Fig. 2 Typical sensor layout for an SHM system

SYSTEM IDENTIFICATION

Normally, a prediction model used for design purposes is thought to be a good representation of the behaviour of the real structure. Design according to the prediction model normally results in the required safety level of the structure. For some cases, however, one needs to know the actual state of a structure in terms of safety. This could be the case for a number of conditions ranging from design verification of structures that are very sensitive to the real dynamic characteristic of the structure or verification of structures that have been exposed to changes during the lifetime of the structure. The changes could be due to a number of reasons such as changes in subsidence, scour, topside weights, corrosion, fatigue cracks or other material degradation.

A number of different methods for system identification are adopted. The choice of the optimal method is dependent on the specific type of structure considered. The preferred method today is SSI as presented in [6]. For other methods used reference can be made to [7], [8], [9] and [10]. Most of the methods referred to commonly can fit under the group of Operational Modal Analysis (OMA) methods.

The only measurements required for system identification based on SSI are the displacements $X(t)$ as measured by the SHM system in a few selected locations on the Topside of the Jacket structure. Measurement of the loading $f(t)$ is not required for extraction of the modal parameters. The modal parameters can be extracted continuously for constant monitoring of structural changes in real-time. For clarity, the modal parameters determined from linear system identification will be referred to as 'experimental modal parameters' to distinguish them from the 'analytical modal parameters' represented by the FE model.

State Space based equations have also been used for other purposes than system identification of modal parameters. In reference [11] and [12] the application of Kalman filters to be used for wave load identification is presented. The advantage of Kalman filter based methods is that these methods allow for analysis of both linear and some nonlinear system behaviour. In addition, they offer the possibility to quantify both measurement uncertainties and prediction model uncertainties from measured and predicted data.

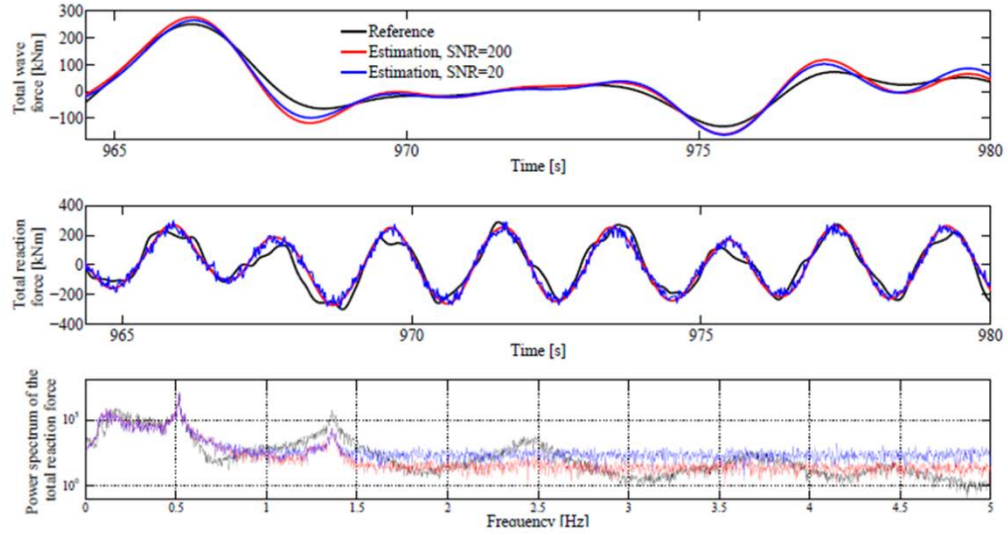


Fig. 3 Example of wave load identification using Kalman filter. Top figure show wave force estimated by Kalman filter versus reference total wave loading force acting on an offshore structure in the time domain. Middle figure show the same, but for the structure reaction forces, instead of total load forces. Bottom curve show the results in the frequency domain. Curves are shown for high and low measurement noise levels.

FEM UPDATING OF MODAL PARAMETERS

The typical FE model input parameters that influence the modal parameters are: mass, centre of gravity (CoG), element stiffness, Local Joint Stiffness (LJF), soil stiffness, damping properties, etc.

Several methods for FEM updating are adopted. Here, for the first FEMU step, the Bayesian Parameter Estimation algorithm is presented. In [13] a more detailed description of the method is presented for updating of FE model against results attained from experimental modal analysis (EMA) in terms of updating to the Frequency Response Function (FRF). Typically, measured FRFs are not available for offshore structures; therefore, the presented method concerns an update with respect to parameters identified by OMA.

The input parameters required for FEM updating are:

- From SHM system: Modal parameters from system identification.
- From FEM model: Predicted modal parameters from FEM.
- From FEM model: Sensitivity matrix for the selected parameters to be updated (mass, COG, stiffness's, etc.).

The updating is performed using the Bayesian Parameter Estimator algorithm:

$$\{\Delta P\} = ([C_p] + [S]^T[C_R][S])^{-1}[S]^T[C_R]\{\Delta R\}. \quad (1)$$

Where:

- $\{\Delta P\}$: Resulting parameter corrections (updated FEM input parameters).
- $\{\Delta R\}$: Response difference (difference between predicted and measured modal parameters).
- $[C_p]$: Confidence matrix for parameters to be updated (uncertainty of FEM input parameters).
- $[C_R]$: Confidence matrix for response (uncertainty of modal parameters).
- $[S]$: Sensitivity matrix, produced through perturbation.

The result from the FEM updating is that the analytical modal parameters now are close to the experimental modal parameters. This is the crucial step in generating a True *Digital Twin* of the real structure. In case of nonlinear system behaviour, the Bayesian Estimator method allows for multi-FEM updating. The issue will not be discussed here.

An example outlining the method in principle for Markov Chain Monte Carlo (MCMC) based FEM updating for offshore structures is presented in [10]. The advantages of the MCMC based methods include more advanced

estimation of both measurement uncertainties and model uncertainties.

During the process of FEMU, valuable information for the design engineer is gained from the generation of the sensitivity matrix. Important information about the characteristics of a structure can be observed. Typically, it can be observed that the natural frequencies are very sensitive to Young's modulus, whereas the mode shapes are not. However, as the prior uncertainty of the Young's modulus is very low, it is typically not changed during an FEM updating process. Updating of masses typically only effects the natural frequencies, but not the mode shapes. Conversely, CoG for masses can affect both natural frequencies and mode shapes (especially in torsional modes).

An interesting trend for FEMU of offshore structures is that the soil/foundation parameters commonly affect the natural frequencies, but not the first two bending mode shapes (two horizontal directions). It does, however, have a significant effect on the torsional mode shapes. This fact typically allows for direct calibration of the soil/foundation properties.

EXPANSION PROCESS

The purpose of the expansion process is to determine stresses and strains in all elements and joints of the structure above and below the Mean Sea water Level (MSL) from displacement measurements of the structure in only a limited number of sensor locations above MSL. Typically, for ease of installation, the sensors are mounted on the Topsides structure.

For the simplest case, the System Equivalent Reduction Expansion Process (SEREP) method can be used for expanding the data [14]. As the name indicates, the SEREP method originally was developed for reduction of the number of degrees of freedom when comparing analytical and measured data. However, the method here is adopted for expansion purposes.

The basic equation for expansion of the measured displacements, $X(t)$ can be performed by multiplying the updated analytical mode shapes, Φ with the modal coordinates $q(t)$:

$$X(t) = \Phi \cdot q(t) \quad (2)$$

The modal coordinates, $q(t)$ are computed independently at each time step t , through a least-squares solution. Strain (and analogue stresses) are now calculated from the determined modal coordinates, $q(t)$:

$$\varepsilon(t) = \Phi_\varepsilon \cdot q(t) \quad (3)$$

where the mode shapes, Φ_ε are now expressed in terms of normalised strain/stress values calculated from the updated FE model, instead of normalised displacements.

As an alternative to modelling dynamic system behaviour by modal analysis (eigenvalue vectors), in some cases dynamic analysis can be performed by use of Load-Dependent Ritz vectors (LDR). For a description of the LDR method, reference is made to e.g. [15]. The advantage of the LDR method is that both the static (quasi-static) behaviour and the dynamic modes shapes can be analysed. One short-coming of the LDR can be the approximate solutions attained for the dynamic mode shapes.

In reference [16], an example of an adopted method for expanding both in the dynamic mode shapes and in the quasi-static Ritz vectors (deflection mode shapes) is described for an experimental laboratory test of an offshore platform - Valdemar VBA. The initial methodology relies on splitting the measured response signal in a low pass filtered (LP) and a high pass filtered (HP) response signal. An alternative approach is through the use of wavelet filters. Ramboll uses Wavelet filters for different purposes ranging from removal of unwanted contributions from natural frequency responses in laboratory wave loading experiment tests to special identification of nonstationary processes.

Fig 3 and 4 show an example of expansion results from an offshore jacket structure where the expansion successfully recreates measured behaviour.

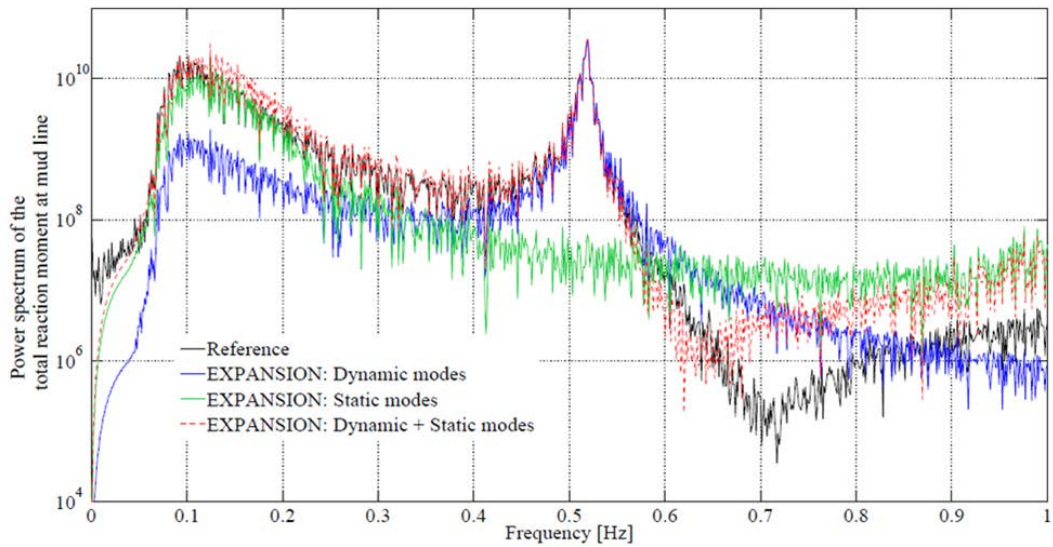


Fig. 4 Expansion showing the results from using only dynamic modes in the expansion (blue), using only the quasi-static deformation modes (green) and using both the dynamic modes and the quasi-static modes in the expansion (red). The measured data is shown as the black curve. It is seen that it is important to expand both in the quasi-static and the dynamic modes.

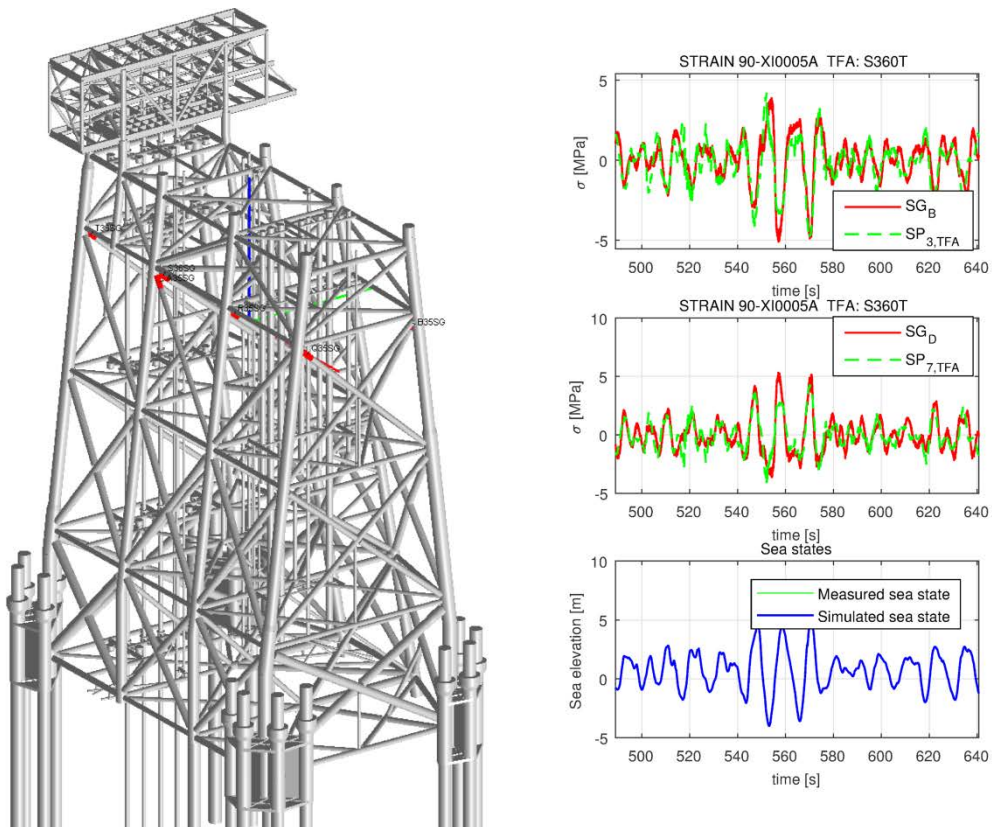


Fig. 5 Example of expansion of strains/stresses in braces in the splash zone of a jacket structure from data from only a few sensors at the Topside (incl. wave radars) and compared to strains/stresses measured directly with strain gauges (blind strain gauges). Even the complicated in-plane and out-of-plane local member bending moment phenomena are correctly expanded

FEM UPDATING OF WAVE LOADING

Having updated the structural properties of the FE model to more closely match the observed modal parameters from the OMA process, the strain predictions from the expansion process are used to calibrate the

wave loading parameters.

The key parameters in the wave loading model are the Morison equation coefficients (C_d, C_m), which mediate the levels of force due to added inertia and added damping from the wave loading. Two procedures for this FEMU step are used specific to individual model requirements, the first being an identical update procedure to that presented for FEMU of the structural parameters in relation to the model properties of the structure. That is, a Bayesian formulation is adopted where prior belief with regards to the parameters is combined with sensitivity analysis to update the most sensitive and least certain parameters.

Alternatively, since there are few parameters to update and prior knowledge is relatively weak, in certain cases a simpler optimisation algorithm is adopted. A variety of optimisation methods are available, the reader is directed to [17] or [18] for good introductory texts.

The optimisation is framed as the minimisation of an objective function. The objective function is chosen to minimise the Bias introduced to the model and also to minimise the uncertainty (at this stage in terms of CoV), with respect to the transient fatigue analysis. The objective function chosen is shown below with α being the function to minimise.

$$\alpha = 1 - \text{Bias}(C_d, C_m) + \text{CoV}(C_d, C_m) \quad (4)$$

The objective function is minimised under the following constraints.

$$Kd = r(C_d, C_m), \quad \text{Bias} \geq 1, \quad C_d > 0, \quad C_m > 0, \quad (5)$$

where Kd is the FE model analysis (e.g. transient fatigue analysis) to be solved as a function of the wave load parameters during the optimisation.

The Bias and Coefficient of Variation (CoV) are calculated for the ratios of measured and predicted stress ranges as a function of Rain Flow Count (RFC) bins for the full measurement period. The purpose of the wave load calibration is to minimize the Bias, i.e. arrive at a Bias value as close as possible to 1.0. Bias values below 1.0 mean a conservative result, while Bias values above 1.0 will not be allowed as a result, i.e. this being a non-conservative calibration. At the same time, the CoV is minimised. The CoV value in this concept is related to the uncertainty of the prediction model.

The optimisation is also constrained to realistic physical values for the parameters and to improve computation speed such that, C_d lies in the range e.g. (0.8, 1.5) and C_m in (1.4, 2.0). The solution to this optimisation problem allows a more accurate representation of the wave loading experienced by the structure which reduces uncertainty in the amount of fatigue the structure has experienced.

A summary of some of the results obtained in the wave loading FEMU step can be seen in Fig. 5, 6, 7. In Fig. 5 the sensitivity to the parameters (C_d, C_m) for each member informs the update step and the improvements from the updated values are also shown.

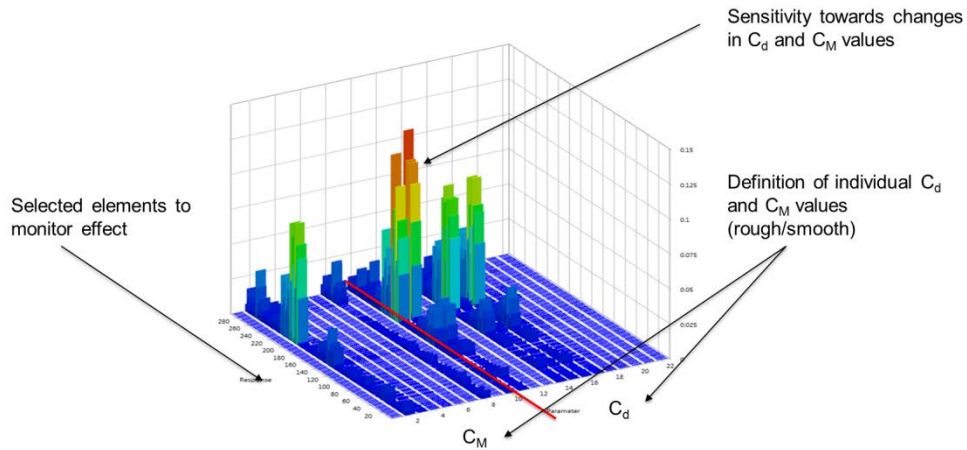


Fig. 6 Generation of the sensitivity matrix as part of the Bayesian based FEM updating of the wave loading parameters. The sensitivity matrix shows which parameters that are dominant and which parameters do not need to be part of the calibration.

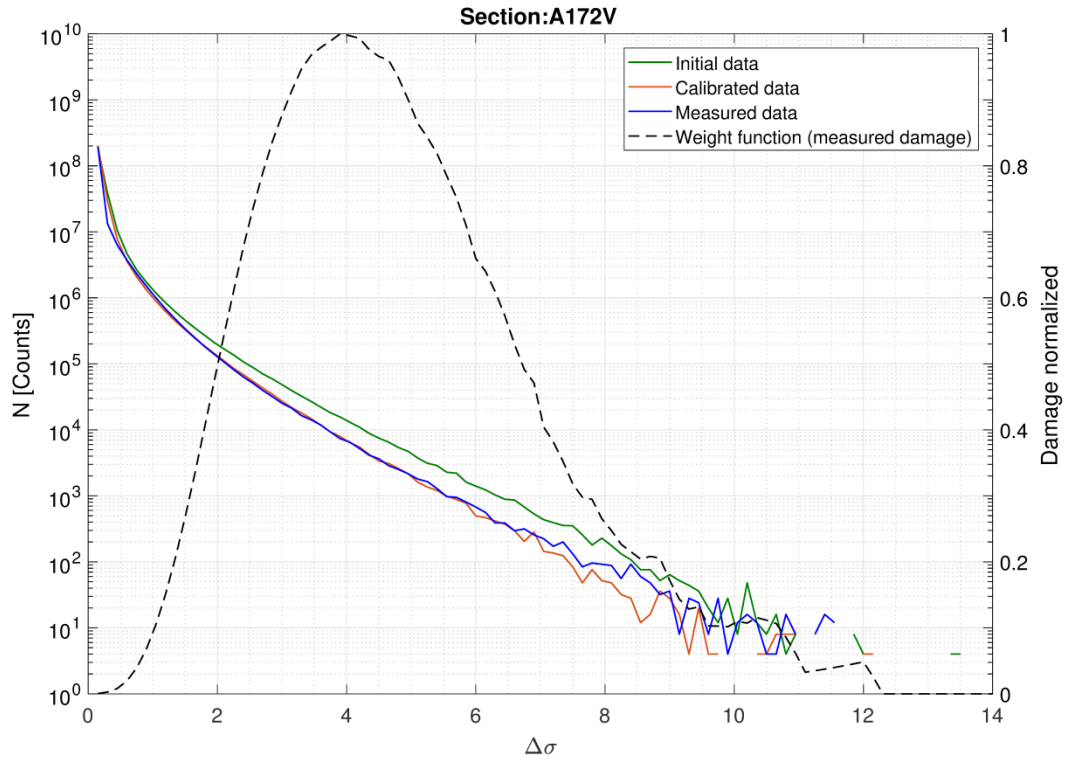


Fig. 7 Results from FEM updating of the wave load parameters in terms of Rain Flow Count stress range history curves, comparing the initial model performance with the updated model performance against the measured curves.

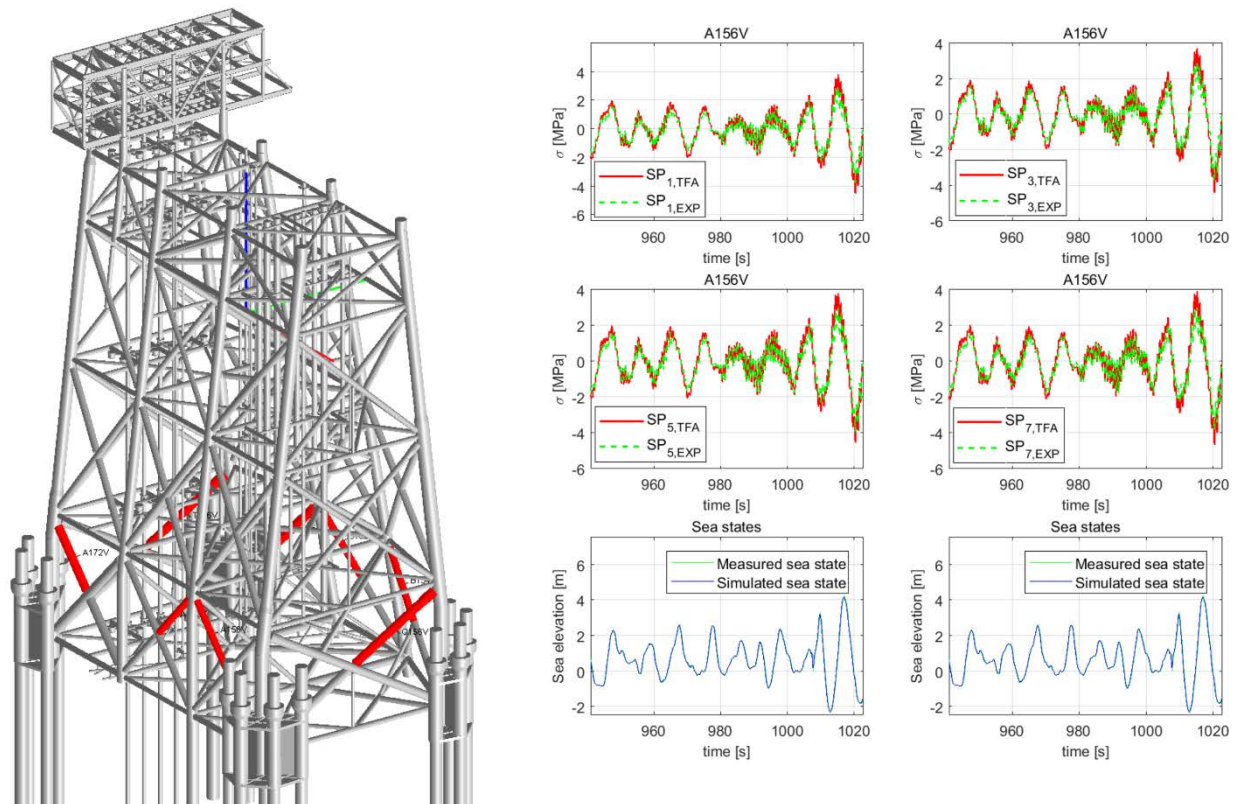


Fig. 8 Curves showing the performance of the updated FE model in term of stresses in the time domain (time histories). The green stress curves are the stresses expanded in the bottom braces of the Jacket based on displacement data measured at the Topside in a few locations. The red stress curves are the stresses in the bottom braces calculated by the updated FE model with measured wave load input from wave radars.

DISCUSSION

This paper has outlined a number of the steps in Ramboll's process for predictive maintenance. The main focus of this paper has been on the creation of a validated model of the structure, including an estimate of the wave loading, updated with information from the measured sea state.

Not presented above, are the next stages in the process, which include quantification of uncertainties and the Risk- and Reliability-based Inspection Planning informed by fatigue calculations based on the validated model. The FEMU of the prediction models results in improvement of the prediction models, but the question remains: by how much?

The purpose of the uncertainty assessment is to quantify how well the improved prediction model performs. The experience from all performed projects up to now is that the uncertainties are reduced by implementation of the presented procedures. Any reduction in uncertainties results in cost reductions for maintenance.

The scope for the uncertainty assessment is from SHM based data to identify and quantify all contributing factors; here follow examples of uncertainties that is quantified:

- Uncertainty from the updated FEM analysis to predict the structural response in terms of element stress ranges when exposed to a measured sea state.
- Uncertainty from the calibration of the FEM analysis when only based on a limited range of measured sea states.
- Uncertainty of the year-to-year sea state variations for short term periods. The term is relevant for measurement periods less than the full design life of the platform. Typically, the value is calculated for a three year period, but any period can be selected. The period is related to the time to first inspection (RBI), not related to the length of the measurement period.
- Uncertainty from measurement noise. The uncertainty covers all kinds of sensors used.
- Uncertainty from the expansion process using an updated FEM to calculate stress ranges for all joints and members in the jacket structure from measurement of responses in only a limited number of locations.

The specific method for quantification of each of the identified contributions from the SHM measurements cannot be presented in the present paper, but will follow in separate papers.

With access to measured structural response and a high-fidelity *True Digital Twin*, online damage detection also becomes more feasible. Ramboll are investigating a number of avenues, including fully data-based approaches [16, 17], as well as those that make inferences from an FE model and the SSI process [18].

FUTURE DIRECTIONS AND CONCLUSIONS

As presented here, the process employed by Ramboll has been found to add value for clients in the offshore industry through the reduction in uncertainty associated with fatigue predictions for the full design life of offshore structures. This has been made possible by careful calibration of FE models via a two-step updating process with reference to measured responses of the structures and sea state observations.

Ramboll are pursuing further improvements to the method, where the stages of the process are more closely integrated with regard to uncertainty propagation. To this end, the incorporation of even more advanced Machine Learning and data-driven SHM techniques to the methodology are currently under development. The present Ramboll approach for quantification of uncertainties from prediction models and SHM measurements is defined so that it is consistent with the approach adopted for Risk- and Reliability-based Inspection Planning as presented in [21]. The introduction of the latest developments within the Machine Learning community may also affect future approaches for inspection planning (RBI).

One promising new area of research is in the use of *Grey-Box* modelling [22]. This development aims to enhance the predictive capability of physics-based (*White-Box*) models (multi-body physics, FEM, etc.) through the addition of Machine Learning components in cases where the physics is not fully understood. A key step in the Ramboll process is the identification of wave loading on a structure, as this impacts not only

the fatigue life but the dynamic behaviour. The first studies on how to improve on the Morison equations for prediction of wave loading by using the latest advances in machine Learning, enhancing the Morison equations by an approach based on Gaussian Process NARX (GP-NARX) models is demonstrated in [24]. The advantage of adapting approaches based on nonlinear system identification methods is not only enhancement of the *White-Box* models, but also of providing natural confidence intervals for predictions and as such facilitates for more detailed information about uncertainties.

In terms of damage detection, the *Grey Box* is also of particular interest for detecting abnormal response from the measurement system. Here, the idea evolves around that of detecting changes in model error and attributing them to a change in the system. In order for this to be robust, one must ensure that the data used are free from outliers and confounding influences from Environmental and Operational Variations (EOV). Here advances in Outlier Analysis and Data Normalisation are promising avenues of investigation [23] and [24].

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REFERENCES

- [1]: OGP Standard Committee. *Reliability of Offshore Structures – Current Design and Potential Inconsistencies*, OGP Report No. 486. International Association of Oil and Gas Producers (OGP, IOGP), March 2014.
- [2]: Ramboll Oil and Gas. *ROSAP, Program ROSA, Structural Analysis, User's Guide*. Ramboll Offshore Structural Analysis Program Package (ROSAP), Rev. 5.1, February 2017.
- [3]: J. B. Hansen, R. Brincker, M. B. Knudsen and U. Tygesen. *Combining GPS and integrated sensor signals*. International Operational Modal Analysis Conference, Istanbul, Turkey, 2011.
- [4]: A. Skafte, U. Tygesen and R. Brincker. *Expansion of Mode Shapes and Responses on the Offshore Platform Valdemar*. International Modal Analysis Conference (IMAC), Orlando, Florida, USA, 2014.
- [5]: E. Dascotte, J. Strobbe and U. T. Tygesen. *Continuous Stress Monitoring of Large Structures*. International Operational Modal Analysis Conference (IOMAC), Guimaraes, Portugal, 2013.
- [6]: R. Brincker and P. Andersen. *Understanding Stochastic Subspace Identification*. Proceedings of the 24th International Modal Analysis Conference (IMAC), St. Louis, Missouri, USA, 2006.
- [7]: B. Peeters, H. Van der Auweraer, P. Guillaume and J. Leuridan. *The PolyMAX frequency-domain method: a new standard for modal parameter estimation?* Shock and Vibration 11 (2004), 395-409, IOS Press.
- [8]: R. Brincker, L. Zhang and P. Andersen. *Modal Identification from Ambient Response using Frequency Domain Decomposition*. Proceedings of the 18th International Modal Analysis Conference (IMAC), San Antonio, Texas, USA, pp.625-630, 2000.
- [9]: L. Zhang, R. Brincker and P. Andersen. *An Overview of Operational Modal Analysis: Major Development and Issues*. Proceedings of the 1st International Operational Modal Analysis Conference (IOMAC), Copenhagen, Denmark, 2005.
- [10]: P. L. Green, U. T. Tygesen and N. Stevanovic. *Bayesian Modelling of Offshore Platforms*. The Society for Experimental Mechanics (SEM), International Modal Analysis Conference (IMAC), Model Validation and Uncertainty Quantification, Orlando, Florida, USA, 2016.
- [11]: N. Perisic, P. H. Kirkegaard and U. T. Tygesen. *Load Identification of Offshore Platform for Fatigue Life Estimation*. International Modal Analysis Conference (IMAC), Orlando, Florida, USA, 2014.
- [12]: N. Perisic and U. T. Tygesen. *Cost-Effective Load Monitoring Methods for Fatigue Life Estimation of Offshore Platform*. Proceedings from the ASME 2014 33rd International Conference on Ocean, Offshore and Arctic Engineering (OMAE), San Francisco, California, USA, 2014.

- [13]: T. Lauwagie, J. Guggenberger, J. Strobbe and E. Dascotte. *Model Updating Using Operational Data*. International Conference on Noise and Vibration Engineering (ISMA), Leuven, Belgium, 2010.
- [14]: J. O'Callahan, P. Avitabile, R. Riemer. *System Equivalent Reduction Expansion Process (SEREP)*. Proceeding of the 7th International Modal Analysis Conference (IMAC), 1989, pp. 29-37.
- [15]: H. Sohn and K. H. Law. *Extraction of Ritz Vectors from Vibration Test Data*. Mechanical Systems and Signal Processing 15, 231-226, 2001.
- [16]: A. Skafte, J. Kristoffersen, J. Vestermarck, U. T. Tygesen and R. Brincker. *Experimental Study of Strain Prediction on Wave Induced Structures Using Modal Decomposition and Quasi Static Ritz Vectors*. Journal Engineering Structures 136, Elsevier, pp. 261-276, 2017.
- [17]: R. Fletcher. *Practical Methods of Optimization*. John Wiley & Sons, 2013.
- [18]: D. Simon. *Evolutionary Optimization Algorithms*. John Wiley & Sons, 2013.
- [19]: M. D. Ulriksen, D. Tcherniak, L. M. Hansen, R. J. Johansen, L. Damkilde and L. Frøyd. *In-situ Damage Localization for a Wind Turbine Blade Through Outlier Analysis of SDDL-induced Stress Resultants*. SAGE Journals, Structural Health Monitoring, 2016.
- [20]: M. D. Ulriksen and L. Damkilde. *Structural damage localization by outlier analysis of signal-processed mode shapes – Analytical and experimental validation*. Mechanical Systems and Signal Processing, Elsevier, 2015.
- [21]: M. Dohler and F. Hille. *Subspace-based damage detection on steel frame structure under changing excitation*. International Modal Analysis Conference (IMAC), Orlando, Florida, USA, 2014.
- [22]: DNVGL-RP-C210. *Probabilistic Methods for Planning of Inspection Planning for Fatigue Cracks in Offshore Structures*. DNV-GL Recommended Practice, Edition November 2015.
- [23]: T. Rogers, G. R. Holmes, E. J. Cross, K. Worden. *On a Grey Box Modelling Framework for Nonlinear System Identification*. Special Topics in Structural Dynamics, Volume 6, pp. 167-178, Springer Link, March, 2017.
- [24]: K. Worden, T. Rogers, E. J. Cross. *Identification of Nonlinear Wave Forces using Gaussian Process NARX Models*. Nonlinear Dynamics, Volume 1, pp. 203-221, Springer Link, May, 2017.
- [25] N. Dervilis, E.J. Cross, R. J. Barthorpe and K. Worden. *Robust methods of inclusive outlier analysis for structural health monitoring*. Journal of Sound and Vibration 333: 5181-5195, 2014.
- [26] E.J. Cross, K. Worden and Q. Chen. *Cointegration; A novel approach for the removal of environmental trends in structural health monitoring data*. Proceedings of the Royal Society A 467 2712-2732 doi:10.1098/rspa.2011.0023, 2011.

